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[illegible] STOCHASTIC DEMAND [illegible] SINGLE MACHINE LOT SCHEDULING PROBLEM

by

[illegible] C. GRAVES

Technical Report No. 140

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A NOTE OF THE DETERMINISTIC DEMAND MULTI-PRODUCT
SINGLE-MACHINE LOT SCHEDULING PROBLEM

by

STEPHEN C. GRAVES

Technical Report No. 140

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FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and industrial grants-in-aid. The work reported herein was supported by the Office of Naval Research under Contract N00014-75-C-0556.

Richard C. Larson
Jeremy F. Shapiro

Co-Directors

ABSTRACT

This note considers two single-machine multi-product scheduling problems with deterministic demand that have appeared in the management science literature: the economic lot scheduling problem and the joint replenishment problem. These problems are shown to be equivalent. In addition, they are shown to be equivalent to a one-warehouse, several-retailer inventory problem for which exists an efficient branch and bound solution procedure.

1. INTRODUCTION

The purpose of this note is to try to unify and extend current theory on production lot sizing for multiple products sharing the same production resource and having known constant demand rates. In particular, two classes of problems, the economic lot scheduling problem (ELSP) and the joint replenishment problem (JRP), are shown to be equivalent. Furthermore, these two problems are shown to be equivalent to the problem of finding single cycle continuous review policies for a one-warehouse, several-retailer inventory system, for which an efficient branch and bound solution procedure exists.

The next two sections define and formulate the economic lot scheduling problem and the joint replenishment problem, respectively. Section 4 shows the equivalence of these two problems, while section 5 equates the two problems to the multi-stage inventory problem.

2. THE ECONOMIC LOT SCHEDULING PROBLEM

The economic lot scheduling problem (ELSP), also known as the single-machine, multi-product lot scheduling problem, assumes a single production facility on which n products are individually produced. The product i is demanded at a constant rate r_i, and, when in production, is produced at a constant rate p_i. A setup cost s_i is incurred whenever production of product i is initiated, and inventory holding costs for product i are linear with respect to the inventory level at rate h_i. All demand is met from inventory with no backordering. The problem is to find a feasible production schedule so to minimize the total setup and inventory holding costs per time unit.

This problem has been considered by several authors. The papers of Rogers [9], Bomberger [1], Madigan [7], Stankard and Gupta [11], and Doll and Whybark [3] are representative of the development of improved theory for this problem. In particular, Doll and Whybark [3], whose procedure is best on a set of test problems, consider policies of the form $(k_1,\ldots,k_n;T)$, where k_i is integer for $i=1,2,\ldots,n$. This policy is a cycling policy with a fundamental period of length T; product i is produced once every k_i periods or every k_iT time units. Note that since production capacity is finite, a given policy $(k_1,\ldots,k_n;T)$ may not be feasible; that is, a production schedule cannot be found such that the production requirements in each period are less than T, the length of the period.

Doll and Whybark present a two-step heuristic procedure for finding a good feasible schedule of the form $(k_1,\ldots,k_n;T)$. The first step is to find a good policy with respect to setup and inventory holding costs, ignoring the question of feasibility. The second step is needed only if the first step produces an infeasible schedule; in this case, the schedule is adjusted until a feasible schedule is found. The problem associated with the first step may be written as the following non-linear, mixed integer math program:

$$
\text{ELSP:}\quad (1)\quad \min \sum_{i=1}^{n} c_i
$$

$$
\left.\begin{array}{lll}
(2)\ \text{s.t.}\ c_i & = & s_i/T_i + \hat{h}_i r_i T_i/2 \\
(3)\quad T_i & = & k_i T \\
(4)\quad \hat{h}_i & = & h_i(1 - r_i/p_i) \\
(5)\quad k_i & \geq & 1, \text{ integer}
\end{array}\right\} \quad i=1,\ldots,n
$$

$$
(6)\quad T \geq 0
$$

where c_i is the average setup cost and holding cost for product i. Doll and Whybark find a heuristic solution to (1) - (6) by means of an iterative procedure. Given values for the vector $(k_1,\ldots,k_n)$, the optimal period length T is easily found; given a value for T, optimal integer values for k_i, $i=1,\ldots,n$, can be found. The procedure starts with $k_i = 1$, for $i=1,\ldots,n$, and stops upon convergence.

3. THE JOINT REPLENISHMENT PROBLEM

The joint replenishment problem (JRP), or packaging problem is identical to the ELSP except in the assumptions concerning the setup costs. For the JRP, a major setup cost S is incurred whenever the production facility is setup after having been idle or shutdown. In addition to this setup cost, a minor setup cost s_i is incurred prior to the processing of product i. The typical scenario for this problem is a facility dedicated to one product which may be packaged into several different containers. The major setup corresponds to the preparing of the facility for production, while the minor setups correspond to the cost of changing the packaging mechanism. Note here that "multiple" products may be the same product in different packages.

Representative of the development of work on this problem are the papers of Shu [10], Nocturne [8], and Goyal [4], [5]. Goyal considers policies of the form $(k_1,\ldots,k_n;N)$ where N is the frequency of major setups for manufacturing runs, and item i is packaged once every k_i manufacturing runs where k_i is integer. This policy class is identical to that considered for the ELSP where $T = 1/N$. Consequently, taking $T = 1/N$, the JRP may be written as

$$\text{JRP: } (1') \quad \min \sum_{i=1}^{n} c_i + S/T$$

$$\text{s.t.} \quad (2) - (6)$$

That is, the JRP is identical to (1) - (6) with only the addition to the objective function of a term for the major setup cost.

Goyal [5] solves (1') - (6) by means of a limited explicit enumeration. Bounds are placed on the optimal value of N (or equivalently T), which yield bounds on the values of k_i. Each possible schedule is then evaluated, with the lowest cost policy being optimal. Goyal assumes that the production rate is such that any policy can be made feasible; hence there is no need to adjust the schedule generated from (1') - (6).

4. EQUIVALENCE OF ELSP AND JRP

To show that the ELSP is equivalent to the JRP it is convenient to rewrite (1) - (6) by substituting (3) into (2), and (2) into (1) to obtain

$$\text{ELSP:} \quad (7) \quad \min \sum_{i=1}^{n} \left(\frac{s_i}{k_i T} + \frac{\hat{h}_i r_i k_i T}{2}\right)$$

$$\left.\begin{array}{lll} (8) \quad \text{s.t.} & \hat{h}_i & = h_i(1 - r_i/p_i) \\ (9) & k_i & \geq 1, \text{ integer} \end{array}\right\} \quad i=1,2,\ldots,n$$

$$(10) \qquad T \geq 0$$

Similarly, by regarding the major setup as an additional product with no demand or production requirements, the JRP is rewritten as

$$\text{JRP:} \quad (7') \quad \min \sum_{i=1}^{n+1} \left(\frac{s_i}{k_i T} + \frac{\hat{h}_i r_i k_i T}{2}\right)$$

$$\text{s.t.} \quad (8), (9), (10)$$

$$(11') \qquad k_{n+1} = 1$$

$$(12') \qquad s_{n+1} = S$$

$$(13') \qquad \hat{h}_{n+1} r_{n+1} = 0$$

That is, product n+1 is setup at cost S during each production period, but has no production requirements. Formulated in this manner, these two problems are identical except that the JRP requires the definition of an additional surrogate product by (11') - (13') to reflect the major setup cost.

5. EQUIVALENCE TO A MULTI-STAGE INVENTORY PROBLEM

Consider a multi-stage inventory problem (MSIP) consisting of one central warehouse which supplies n-1 independent retailers. Demand occurs only at the retailers and is deterministic and constant over time; all delivery lead times are assumed to be zero. A fixed ordering cost K_i is incurred whenever retailer i places an order on the warehouse for $i=1,\ldots,n-1$, and an ordering cost K_n is incurred whenever the warehouse (stage n) orders from its external supplier. At each stage, a linear inventory holding cost c_i is charged against the echelon inventory at that stage, as defined by Clark and Scarf [2]. All demand is met from inventory with no backordering. The problem is to find a continuous review inventory policy to minimize total setup and inventory holding costs per unit time.

Graves and Schwarz [6] have analyzed the MSIP over the class of stationary single-cycle policies. A stationary policy assumes that each stage orders the same lot size whenever it orders, with lot sizes allowed to vary across stages. A single-cycle policy assumes that, ignoring any fixed lead times or delivery times, whenever the warehouse orders, all retailers also order. When lead times are present, this class of policies may be restated to reflect the time offset created by the deterministic lead times. To determine the optimal stationary single-cycle policy, the following problem is solved:

MSIP: (14) $$\min \sum_{i=1}^{n} \left(\frac{K_i D_i}{Q_i} + \frac{c_i Q_i}{2}\right)$$

(15) $$\text{s.t.} \quad \frac{Q_n}{D_n} = \frac{k_i Q_i}{D_i}, \qquad i=1,2,\ldots,n-1$$

(16) $$k_i \geq 1, \text{ integer}, \qquad i=1,2,\ldots,n-1$$

(17) $$Q_n \geq 0$$

where D_i is the demand rate at retailer i for $i=1,\ldots,n-1$, $D_n = \sum_{i=1}^{n-1} D_i$ is the demand rate at the warehouse, Q_i is lot size at stage i, and k_i is the frequency of orders at retailer i relative to orders by the warehouse. Hence the optimal solution to (14) - (17) may be characterized by the vector $(k_1,\ldots,k_{n-1};Q_n)$, which implies that the warehouse orders a lot size of Q_n every time it orders, and retailer i orders k_i times between successive orders by the warehouse.

To show the equivalence of the MSIP to both the ELSP and the JRP, it is necessary to define $T = D_n/Q_n$, the frequency with which the warehouse orders. By substitution, (14) - (17) may be restated as

MSIP: (18) $\min \sum_{i=1}^{n} (K_i k_i T + \frac{c_i D_i}{2k_i T})$

(19) s.t. $k_i \geq 1$, integer $\quad i=1,2,\ldots,n-1$

(20) $k_n = 1$

(21) $T \geq 0$

The problem (18) - (21) is equivalent to both (7) - (10) and (7') - (13') where $K_i = \hat{h}_i r_i/2$ and $c_i D_i/2 = s_i$.

In [6], Graves and Schwarz develop and demonstrate a very efficient branch and bound procedure for optimally solving (18) - (21). Clearly this procedure is now applicable to both the ELSP and the JRP. In comparison with the procedure of Doll and Whybark for the ELSP, the branch and bound procedure finds optimal solutions; in comparison with Goyal's procedure for the JRP, the branch and bound procedure is implicit enumeration, rather than explicit, and hence is more efficient.

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